

# Realistic Shell-Model Calculations for $^{208}\text{Pb}$ Neighbors

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**Abstract.** We have performed a shell-model study of the two nuclei  $^{210}\text{Po}$  and  $^{206}\text{Hg}$ , having and lacking two protons with respect to doubly magic  $^{208}\text{Pb}$ . In our calculations we have employed realistic effective interactions derived from the Bonn A nucleon-nucleon interaction. The calculated results are compared with the available experimental data which are, however, very scanty for  $^{206}\text{Hg}$ . The very good agreement obtained for  $^{210}\text{Po}$  supports confidence in our predictions for  $^{206}\text{Hg}$ .

## 1 Introduction

The region of nuclei around  $^{208}\text{Pb}$  has long been the subject of both experimental and theoretical studies. Clearly, this is related to the fact that  $^{208}\text{Pb}$  is a very good doubly magic nucleus, so that the structure of neighboring nuclei, having or lacking nucleons with respect to it, can be appropriately described in terms of shell model.

In this work, we focus attention on the  $N = 126$  isotones  $^{210}\text{Po}$  and  $^{206}\text{Hg}$ , since nuclei with two valence particles or holes provide an ideal testing ground for the matrix elements of the two-body residual interaction. In most of the calculations performed so far for these nuclei [1, 2] empirical effective interactions have been used. As early as some twenty-five years ago, however, a realistic effective interaction, derived from the Hamada-Johnston nucleon-nucleon ( $NN$ ) potential [3], was employed in the works of Refs. [4, 5, 6] to calculate two-particle and two-hole states in the Pb region. Since that time there has been substantial progress towards a microscopic approach to shell-model calculations starting from a free  $NN$  potential. This has concerned both the two basic ingredients involved in such an approach, namely the  $NN$  potential and the many-body methods for deriving the model-space effective interaction. These improvements have been incorporated into the present calculations, which are a part of an extensive study aimed at understanding the role of modern realistic interactions in the shell-model approach to the nuclear many-body problem [7, 8, 9, 10]. More precisely, our effective interaction has been derived from the meson-theoretic Bonn A potential within the framework of a  $G$ -matrix folded-diagram method.

The paper is organized as follows. In Sec. II we give a brief description of our calculations. In Sec. III we present the results obtained for  $^{210}\text{Po}$  and  $^{206}\text{Hg}$  and compare them with experimental data. In Sec. IV we draw some conclusions of our study.

## 2 Outline of Calculations

As already mentioned in the Introduction, we make use of a realistic effective interaction derived from the Bonn A free  $NN$  potential. This was obtained using a  $G$ -matrix folded-diagram formalism, including renormalizations from both core polarization and folded diagrams. Since the valence-proton and -neutron orbits outside  $^{208}\text{Pb}$  are different, we have chosen the Pauli exclusion operator  $Q_2$  in the  $G$ -matrix equation,

$$G(\omega) = V + VQ_2 \frac{1}{\omega - Q_2 T Q_2} Q_2 G(\omega) , \quad (1)$$

as specified by  $(n_1, n_2, n_3) = (22, 45, 78)$  for the neutron orbits, and by  $(n_1, n_2, n_3) = (16, 36, 78)$  for the proton orbits [11]. Here  $V$  represents the  $NN$  potential,  $T$  denotes the two-nucleon kinetic energy, and  $\omega$  is the so-called starting energy. We employ a matrix inversion method to calculate the above  $G$  matrix in an essentially exact way [11, 12]. The effective interaction  $V_{\text{eff}}$ , which is energy independent, can be schematically written in the operator form as

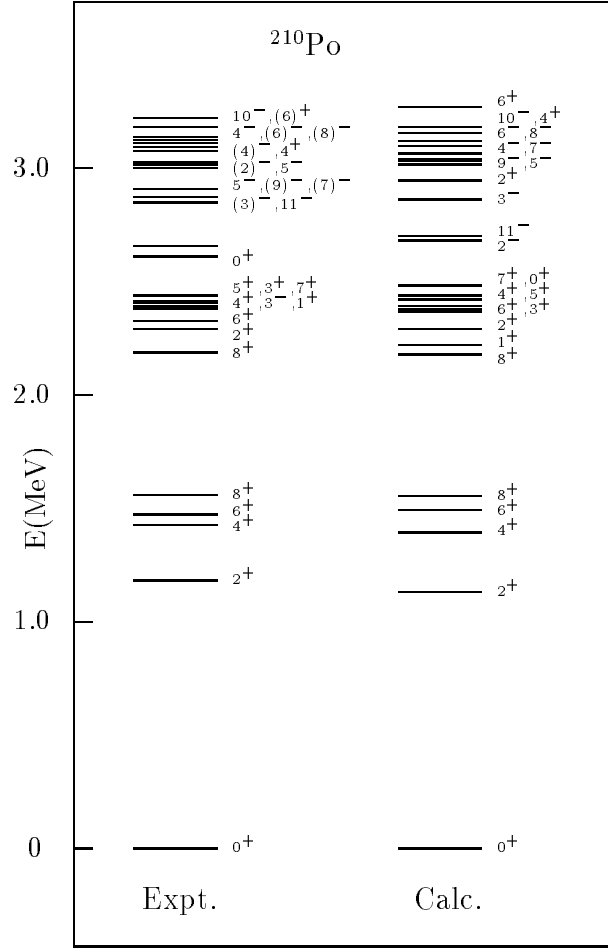
$$V_{\text{eff}} = \hat{Q} - \hat{Q}' \int \hat{Q} + \hat{Q}' \int \hat{Q} \int \hat{Q} - \hat{Q}' \int \hat{Q} \int \hat{Q} \int \hat{Q} + \dots , \quad (2)$$

where  $\hat{Q}$  and  $\hat{Q}'$  represent the  $\hat{Q}$  box, composed of irreducible valence-linked diagrams, and the integral sign represents a generalized folding operation. We take the  $\hat{Q}$  box to be composed of  $G$ -matrix diagrams through second order in  $G$ ; they are just the seven first- and second-order diagrams considered by Shurpin *et al.* [13]. It should be mentioned that in  $^{206}\text{Hg}$  we treat protons as valence holes, which implies the derivation of a hole-hole effective interaction. In the calculation of  $V_{\text{eff}}$  we use an isospin uncoupled representation, where protons and neutrons are treated separately. For the shell-model oscillator parameter we have used 6.88 MeV, as obtained from the expression  $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$  for  $A = 208$ . A detailed description of our derivation including more references can be found in Ref. [8].

As regards the single particle energies, we have taken them from the experimental spectra of  $^{209}\text{Bi}$  and  $^{207}\text{Tl}$  [14, 15]. Thus, for  $^{210}\text{Po}$  we have used the following values (in MeV):  $\epsilon_{h_{9/2}} = 0.0$ ,  $\epsilon_{f_{7/2}} = 0.896$ ,  $\epsilon_{i_{13/2}} = 1.609$ ,  $\epsilon_{f_{5/2}} = 2.826$ ,  $\epsilon_{p_{3/2}} = 3.119$ ,  $\epsilon_{p_{1/2}} = 3.633$ , while for  $^{206}\text{Hg}$  the adopted single-hole spectrum is  $\epsilon_{s_{1/2}} = 0.0$ ,  $\epsilon_{d_{3/2}} = 0.351$ ,  $\epsilon_{h_{11/2}} = 1.348$ ,  $\epsilon_{d_{5/2}} = 1.683$ ,  $\epsilon_{g_{7/2}} = 3.474$ .

## 3 Results

In Fig.1 we report all the experimental [16, 17] and calculated levels of  $^{210}\text{Po}$  up to about 3.2 MeV.

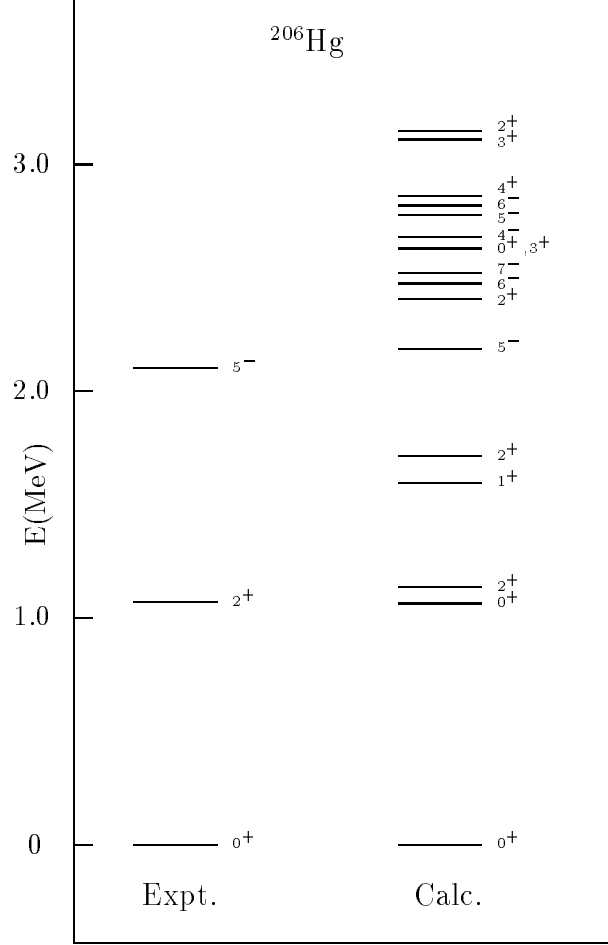


**Fig. 1.** Experimental and calculated spectrum of  $^{210}\text{Po}$ .

We see that each state of a given  $J^\pi$  in the calculated spectrum has its experimental counterpart, the only exception being the  $2_3^+$  state. Experimentally, however, two levels with no angular momentum assignment have been observed at 2.658 and 2.872 MeV. One of these states may correspond to the theoretical  $2_3^+$  at 2.947 MeV. Three of the reported experimental levels, namely the  $3_1^-$  state at 2.387 MeV, the  $5_1^-$  state at 2.910 MeV, and the  $4_1^-$  state at 3.112 MeV, cannot be described within our model space; the first one reflects the collective nature of the octupole  $3^-$  state at 2.615 MeV in  $^{208}\text{Pb}$ , while the other two levels arise from the neutron particle-hole configuration  $\nu(g_{9/2}p_{1/2}^{-1})$  [17]. A measure of the quality of the results is given by the rms deviation  $\sigma$  [18], whose value relative

to the 25 identified excited states is 92 KeV.

In Fig. 2 the observed [19] and theoretical spectra of  $^{206}\text{Hg}$  are reported.



**Fig. 2.** Experimental and calculated spectrum of  $^{206}\text{Hg}$ .

Only three excited states have been observed in this nucleus, and one of them, the  $0_2^+$  at 3.625 MeV, is recognized to be a neutron pairing vibration [20]. For this reason we have not reported this level in Fig.1.

We have also calculated the ground-state binding energies relative to  $^{208}\text{Pb}$ . As for the Coulomb energy, we have taken that of a homogeneous charged sphere, whose radius is  $R = r_0 A^{1/3}$ , with  $r_0 = 1.2$  fm. We find  $E_b(^{210}\text{Po})=8.871$  and  $E_b(^{206}\text{Hg})=-15.165$  MeV, to be compared with the experimental values [21]

$8.783 \pm 0.004$  and  $-15.382 \pm 0.021$  MeV, respectively.

**Table 1.** Calculated and experimental reduced transition probabilities (in W.u.). The experimental data are from [16, 19].

Nucleus	$\lambda$	$J_i^\pi \rightarrow J_f^\pi$	$B(E\lambda)$	
			Calc.	Expt.
$^{210}\text{Po}$	2	$2_1^+ \rightarrow 0_1^+$	3.62	$0.56 \pm 0.12$
	2	$4_1^+ \rightarrow 2_1^+$	4.49	$4.53 \pm 0.15$
	2	$6_1^+ \rightarrow 4_1^+$	3.08	$3.00 \pm 0.12$
	2	$8_1^+ \rightarrow 6_1^+$	1.25	$1.10 \pm 0.05$
	3	$11_1^- \rightarrow 8_2^+$	7.5	$19.7 \pm 1.1$
	3	$11_1^- \rightarrow 8_1^+$	0.53	$3.71 \pm 0.10$
$^{206}\text{Hg}$	2	$2_1^+ \rightarrow 0_1^+$	5.2	$> 0.00027$
	3	$5_1^- \rightarrow 2_1^+$	0.432	$0.182 \pm 0.018$

In Table I the experimental reduced transitions probabilities in  $^{210}\text{Po}$  and  $^{206}\text{Hg}$  [16] are compared with the calculated ones. We have used an effective proton charge  $e_p^{\text{eff}} = 1.5 e$ , which is consistent with the values adopted by other authors [2, 6]. As regards  $g_s$  and  $g_l$ , we have taken the values  $g_s = 3.5$  and  $g_l = 1.12$ , which reproduce the  $g$ -factor of the  $(\frac{9}{2}^-)_1$  state in  $^{209}\text{Bi}$  [14] and the  $B(M1; (\frac{3}{2}^+)_1 \rightarrow (\frac{1}{2}^+)_1)$  in  $^{207}\text{Tl}$  [15]. The theoretical  $B(E2)$  values are in good agreement with the observed ones, except for the  $B(E2; 2_1^+ \rightarrow 0_1^+)$  in  $^{210}\text{Po}$ , which is overestimated by a factor of about six. Our theoretical value, however, is consistent with that obtained by previous calculations [2]. The calculated  $B(E3)$ 's in  $^{210}\text{Po}$  are underestimated with respect to the experimental ones, but they are more sensitive to possible collective core excitations. In fact, all over the trans-lead region the large observed  $B(E3)$  values reflect the collective nature of the  $3^-$  state at 2.615 MeV in  $^{208}\text{Pb}$  [22].

Two quadrupole moments in  $^{210}\text{Po}$  and one in  $^{206}\text{Hg}$  are experimentally known: they are  $Q(8_1^+)$  and  $Q(11_1^-)$  in  $^{210}\text{Po}$ , and the  $Q(5_1^-)$  in  $^{206}\text{Hg}$ . Our calculated values are  $-58.8$ ,  $-97$ , and  $53 \text{ e} \cdot \text{fm}^2$  to be compared with the experimental values [23, 24]  $-55.2 \pm 2.0$ ,  $-86 \pm 11$ , and  $74 \pm 15 \text{ e} \cdot \text{fm}^2$ , respectively.

In Table II we compare the experimental  $g$ -factors [16, 19] with the calculated ones.

**Table 2.** Calculated and experimental  $g$ -factors. The experimental data are from [16, 19].

Nucleus	$J^\pi$	$g$	
		Calc.	Expt.
$^{210}\text{Po}$	$6_1^+$	0.906	$0.913 \pm 0.006$
	$8_1^+$	0.907	$0.919 \pm 0.005$
	$11_1^-$	1.140	$1.108 \pm 0.012$
$^{206}\text{Hg}$	$5_1^-$	1.17	$1.09 \pm 0.01$

## 4 Summary

In summary, we have presented here the results of a shell-model study of the  $N = 126$  isotones  $^{210}\text{Po}$  and  $^{206}\text{Hg}$ , where use has been made of effective two-particle and two-hole interactions derived from the Bonn A  $NN$  potential. The agreement between theory and experiment is very good for both nuclei. The data available on  $^{206}\text{Hg}$  are, however, rather scanty. More experimental information on this nucleus is most desirable to put to a test the predictive power of our calculations. It should be emphasized that, together with those of Ref. [10], these are the first shell-model calculations in the lead region where the effective interaction is derived from a modern  $NN$  potential by means of a  $G$ -matrix folded-diagram method.

In a forthcoming paper we shall present the results of an extensive study of the  $N = 126$  isotones [25]. Here, we conclude that the present results, which are consistent with those obtained in our previous works [7, 8, 9, 10], provide further insight into the role of modern realistic interactions in nuclear structure calculations, evidencing, in particular, the reliability of the Bonn potential.

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## References

1. C. W. Ma and W. True, Phys. Rev. C **8**, 2313 (1973).
2. D. Zwarts and P. W. M. Glaudemans, Z. Phys. A **320**, 487 (1985).
3. T. Hamada and I. D. Johnston, Nucl. Phys. **34**, 382 (1962).

4. T. T. S. Kuo and G. H. Herling, US Naval Research Laboratory Report n. 2258, 1971.
5. G. H. Herling and T. T. S. Kuo, Nucl. Phys. **A181**, 113 (1972).
6. J. B. McGrory and T. T. S. Kuo, Nucl. Phys. **A247**, 283 (1975).
7. F. Andreozzi, L. Coraggio, A. Covello, A. Gargano, T. T. S. Kuo, Z. B. Li, and A. Porrino, Phys. Rev. C **54**, 1636 (1996).
8. A. Covello, F. Andreozzi, L. Coraggio, A. Gargano, T. T. S. Kuo, and A. Porrino, Prog. Part. and Nucl. Phys. **38**, 165 (1997).
9. F. Andreozzi, L. Coraggio, A. Covello, A. Gargano, T. T. S. Kuo, and A. Porrino, Phys. Rev. C **56**, R16 (1997).
10. L. Coraggio, A. Covello, A. Gargano, N. Itaco, and T. T. S. Kuo, Phys. Rev. C **58**, 3346 (1998).
11. E. M. Krenciglowa, C. L. Kung, T. T. S. Kuo, and E. Osnes, Ann. Phys. (N.Y.) **101**, 154 (1976).
12. S. F. Tsai and T. T. S. Kuo, Phys. Lett. **39B**, 427 (1972).
13. J. Shurpin, T. T. S. Kuo, and D. Strottman, Nucl. Phys. **A408**, 310 (1983).
14. M. J. Martin, Nucl. Data Sheets **63**, 723 (1991).
15. M. J. Martin, Nucl. Data Sheets **70**, 315 (1993).
16. E. Browne, Nucl. Data Sheets **65**, 209 (1992).
17. L. G. Mann, K. H. Maier, A. Aprahamian, J. A. Becker, D. J. Decman, E. A. Henry, R. A. Meyer, N. Roy, W. Stöfft, and G. L. Struble, Phys. Rev. C **38**, 74 (1988).
18. We define  $\sigma = \{(1/N_d) \sum_i [E_{\text{expt}}(i) - E_{\text{calc}}]^2\}^{1/2}$ , where  $N_d$  is the number of data.
19. R. G. Helmer and M. A. Lee, Nucl. Data Sheets **61**, 93 (1990).
20. E. R. Flynn, D. L. Hanson, R. V. Poore, S. D. Orbesen, and D. A. Lind, Phys. Lett. **76B**, 197 (1978).
21. G. Audi and A. H. Wapstra, Nucl. Phys. **A565**, 1 (1993).
22. I. Bergström and B. Fant, Phys. Scr. **T31**, 26 (1985).
23. J. A. Becker *et al.*, Nucl. Phys. **A522**, 483 (1991).
24. J. A. Becker, *et al.*, Phys. Rev. C **26**, 914 (1982).
25. L. Coraggio, A. Covello, A. Gargano, N. Itaco, and T. T. S. Kuo, to be published.